Maximal 3–γ–vertex–critical graphs

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Outline

- Introduction
- Preliminaries
- Main results

Introduction

G : finite undirected simple graph

$V(G)$: vertex set of G

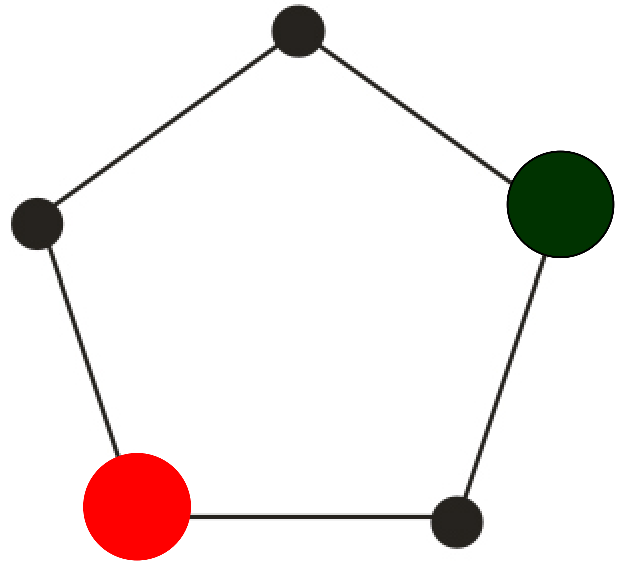
$E(G)$: edge set of G

$\delta(G)$: the minimum degree among the vertices of G

$\overline{G}$: the complement of G

$S \subseteq V(G)$ dominates G if every vertex of G is either in S or adjacent to a vertex in S.

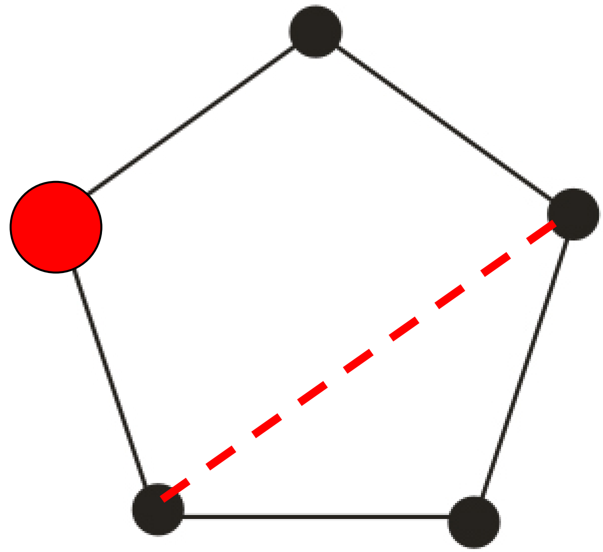
The domination number of G is the cardinality of any smallest dominating set in G and is denoted by $\gamma(G)$.



$$\gamma(C_5) = 2$$

A graph G is **k–γ–edge–critical** if $\gamma(G) = k$, but $\gamma(G + e) < k$, for every edge $e \in E(\overline{G})$.

A graph G is **k–γ–vertex–critical** if $\gamma(G) = k$, but $\gamma(G - v) < k$, for every vertex $v \in V(G)$.

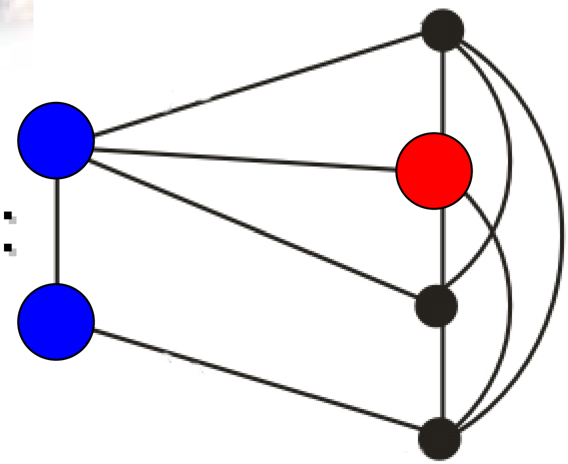


$\gamma(C_5) = 2$

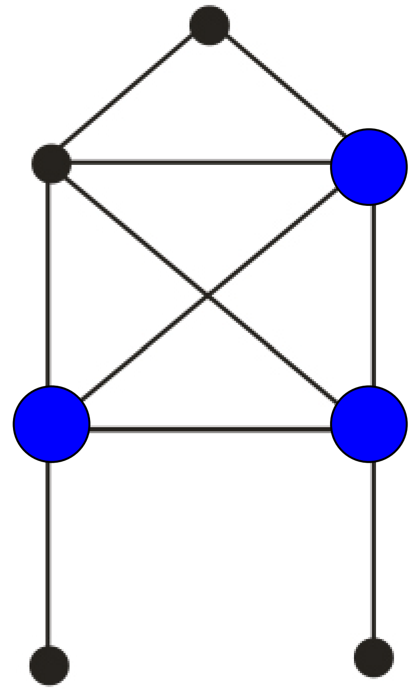
C_5 is not 2–γ–edge–critical.

C_5 is not 2–γ–vertex–critical.

G_1:



G_2:



G_1 is 2–γ–edge–critical.

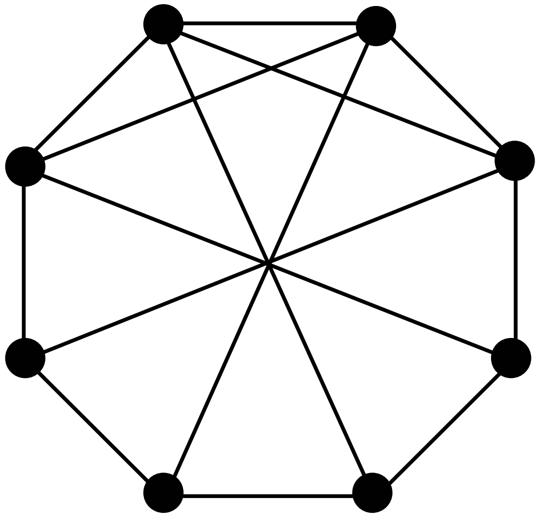
G_2 is 3–γ–edge–critical.

But G_1 is not 2–γ–vertex–critical.

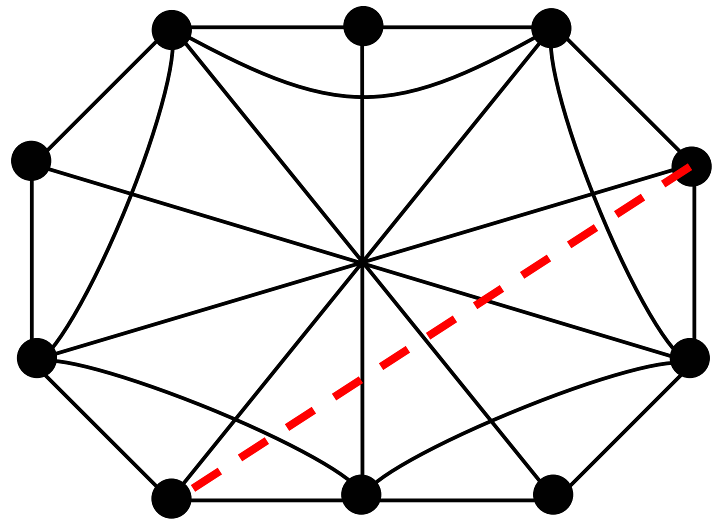
But G_2 is not 3–γ–vertex–critical.

3–γ–vertex–critical graphs.

G_1:



G_2:



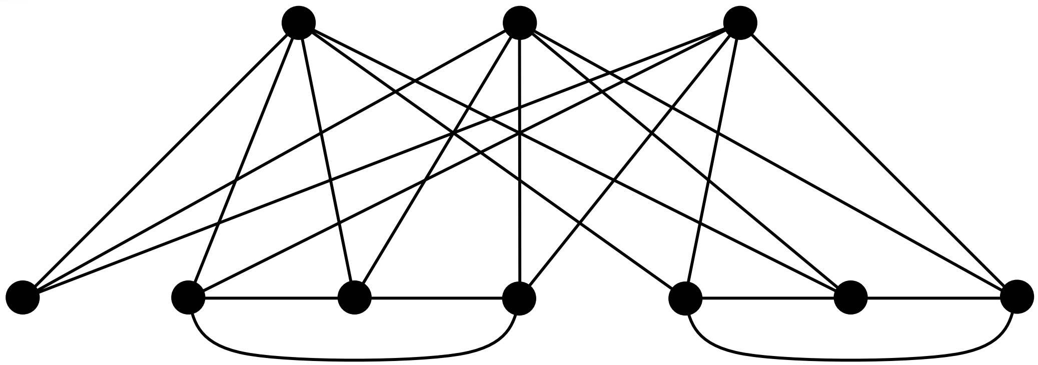
G_1 is 3–γ–edge–critical.

G_2 is not 3–γ–edge–critical.

A connected graph G is maximal k–γ–vertex–critical (m$k$vc) if G is k–γ–vertex–critical and for each $e \in E(\overline{G})$, $\gamma(G + e) < k$.

Observe that G is m$k$vc if and only if G is k–γ–vertex–critical and G is k–γ–edge–critical.

G :



G is 3–γ–vertex–critical.

G is also 3–γ–edge–critical.

G is m3vc.

A characterization of m*k*vc graphs.

k	graphs
1	K_1
2	K_{2n} – a perfect matching, $n \geq 2$.

$k \geq 3$

No characterization of m$k$vc graphs!!!

Preliminaries

Theorem (Sumner and Blitch (1983))

Let G be a connected 3–γ–edge–critical graph. Then

1. If G is of even order, then G has a perfect matching.
2. The diameter of G is at most 3.

Preliminaries (cont.)

Theorem (Paris et al. (1999))

For $k \geq 2$, a k–γ–edge–critical and k–γ–vertex–critical graph is 2–connected.

A graph G is *k*-factor-critical if for every set $S \subseteq V(G)$ with $|S| = k$, the graph $G - S$ contains a perfect matching.

$k = 1$:   factor-critical

$k = 2$:   bicritical

Preliminaries (cont.)

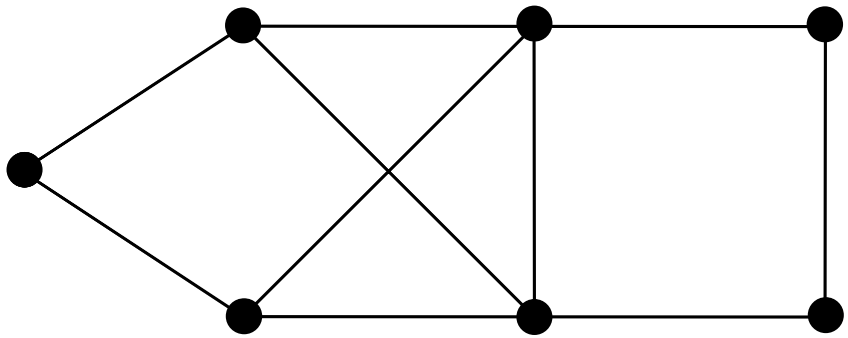
Theorem (Ananchuen and Plummer (2004))

Let G be a 2–connected 3–γ–critical graph of odd order. Then G is factor–critical.

Main results

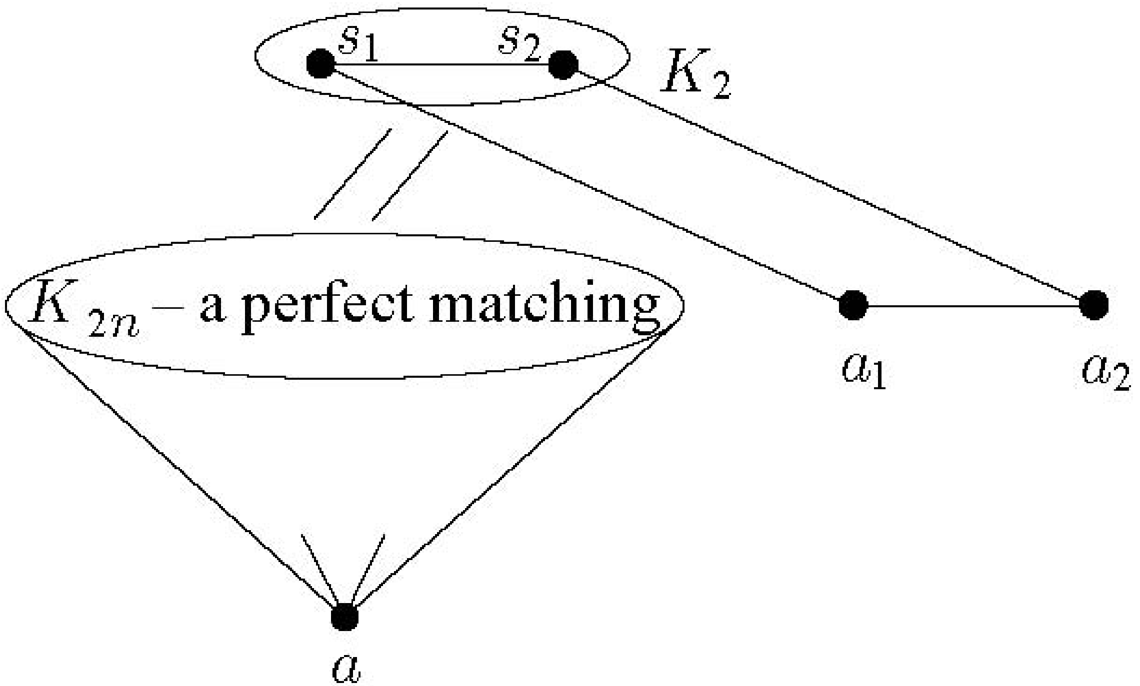
Theorem 1: Let G be a m3vc connected graph.

If $\delta(G) \geq 3$, then G is 3–connected.

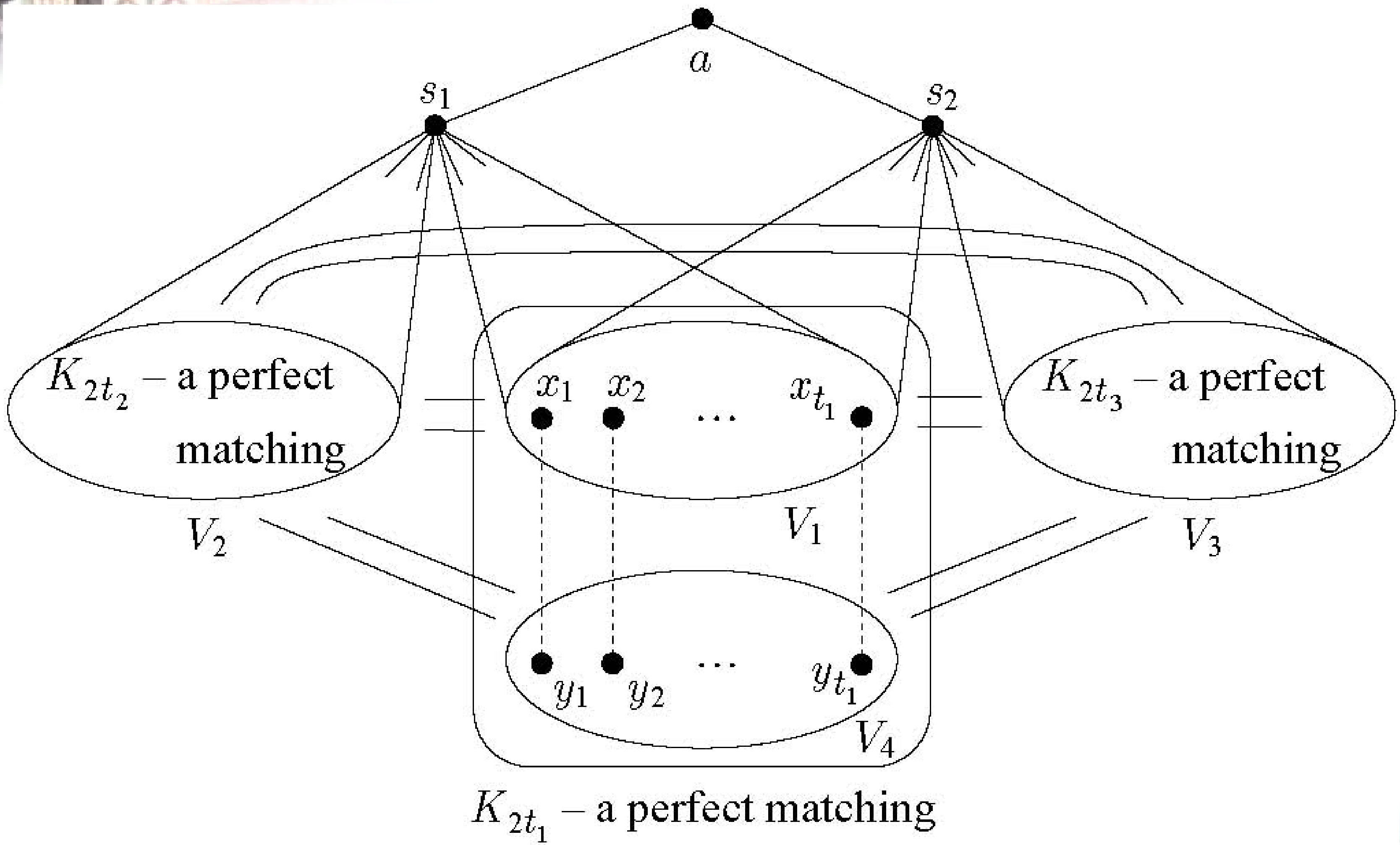


A m3vc graph of connectivity two.

m3vc graphs of connectivity two



The graph H_1



a
s1
s2
K2t2 – a perfect matching
x1 x2 … xt1
K2t3 – a perfect matching
V2
V1
V3
y1 y2 … yt1
V4
K2t1 – a perfect matching

The graph H_2

Note that the graphs H_1 and H_2 are of odd order.

Main results (cont.)

Theorem 2: Let G be a m3vc graph of connectivity two. Then G is isomorphic to either H_1 or H_2.

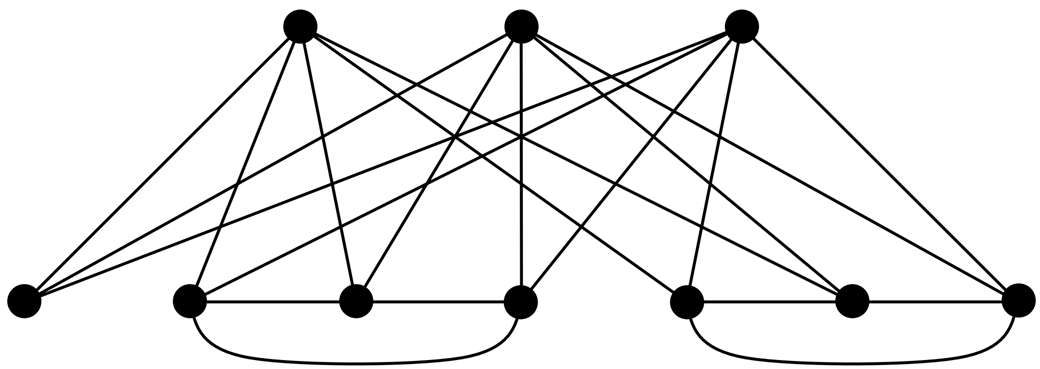
Corollary 3: If G is a m3vc graph of even order, then G is 3–connected.

Main results (cont.)

Corollary 4: A m3vc graph G is factor–critical if G is of connectivity two or odd order.

Main results (cont.)

Theorem 5: Let G be a m3vc graph of even order. Then G is 3–connected bicritical or G is isomorphic to the following graph.



Main results (cont.)

Corollary 6: If G is a m3vc graph of even order at least 12, then G is 3–connected bicritical.

Thank you